

CPT, Lorentz invariance and anomalous clash of symmetries

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Abstract. In this paper we first discuss the analysis regarding the role of Lorentz symmetry in the perturbative non-gravitational anomalies for a family of fermions, which has been recently performed in [arXiv:0809.0184](https://arxiv.org/abs/0809.0184). The theory is assumed to be translational invariant, power-counting renormalizable and based on a local action, but is allowed to have general Lorentz violating operators, including those that break CPT. The main result is that Lorentz symmetry does not participate in the clash of symmetries that leads to the anomalies. Moreover, here we provide a simple *semiclassical* argument that shortly illustrates the origin of this fact.

1. Introduction

Field theories that do not possess Lorentz symmetry have attracted much interest among particle physicists during the last decades. One of the main motivations is the possibility to interpret these models as effective descriptions of more fundamental theories where gravity is consistently included and the breaking of Lorentz invariance occurs spontaneously [1]. Relaxing Lorentz invariance might also open further ways to address phenomenological problems (some examples are provided in [2]).

Among the most popular frameworks, which extend the Standard Model (SM) by incorporating Lorentz violating operators, there are the works by Coleman and Glashow [3] and Colladay and Kostelecky [4, 5], where quite general assumptions, like locality, translational invariance and power-counting renormalizability, are made. This Standard Model Extension (SME) provides a framework where Lorentz symmetry can be tested experimentally and bounds on the Lorentz violating operators can be obtained explicitly. Also such a framework gives us the possibility of investigating conceptual issues related to the role of Lorentz invariance in modern theories of Particle Physics. Much work has been done along these lines. For example causality, stability [6], renormalization [7, 8] and gauge invariance in the extended QED [9] have been studied in the presence of small Lorentz violating perturbations.

In the present paper we first summarize the analysis of Ref. [10]. There a derivation of the perturbative non-gravitational anomalies in the presence of quite general (but small) Lorentz violating operators has been provided, including the violation of CPT. What is the role of Lorentz invariance in the anomalies? Is it possible to relax the standard anomaly cancellation conditions by relaxing Lorentz symmetry? These questions can be reformulated as follows. Are the anomalies the impossibility of defining a quantum *relativistic* theory with simultaneously conserved internal currents or does Lorentz symmetry behave as a spectator in this clash? The

results of Ref. [10] show that the second possibility is the correct one, at least by making the general assumptions of the SME and by assuming the mixing between fermions to be diagonal in the family space. Here we also provide a further argument which leads to the same conclusion, but has the advantage of being relatively simple and short. As we shall see, this reasoning exploits the structure of the classical fermion action in the presence of Lorentz violating parameters and the symmetries of the anomalous Ward identities in standard theories.

2. The fermion sector and general results

In the following we will write an action that contains a certain number of Lorentz violating terms. Let us assume the corresponding parameters to be small (in a sense that will be clarified later on). This assumption allows us to consider the fermion field as an object with four *spinorial* components, as a discrete quantity cannot be changed by a small perturbation. The first part of this section is completely standard and has been introduced to fix the conventions for the subsequent parts. We consider a family of fermions ψ in a general representation of a (compact Lie) group G . We take as part of the definition of family the property that the representation can be made by one or more than one irreducible representations (irreps), but the irreps are all different. The infinitesimal action of G on ψ is

$$\delta\psi = i\Omega\psi \equiv i\Omega^b \left(t_L^b P_L + t_R^b P_R \right) \psi, \quad (2.1)$$

where Ω^b represent the group transformation parameters, $P_{L(R)} \equiv (1 \pm \gamma_5)/2$ is the projector on the left-handed (right-handed) subspaces, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ and $t_{L(R)}^b$ are the hermitian generators in the left-handed (right-handed) representation.

We imagine that each generator of G corresponds to a gauge field A_μ^b , $\mu = 0, 1, 2, 3$, but we do not require each gauge field to be dynamical: in this way we can study both the anomalies associated with gauge currents and those associated with global currents. The action of G on A_μ^b and the covariant derivative of the fermions are respectively defined by

$$\delta A_\mu^b = f^{cdb} \Omega^d A_\mu^c + \partial_\mu \Omega^b, \quad (2.2)$$

where f^{cdb} represents the structure constants of G , satisfying $[t_L^b, t_L^c] = if^{bcd} t_L^d$, and

$$D_\mu\psi \equiv \left[\partial_\mu - iA_\mu^b \left(t_L^b P_L + t_R^b P_R \right) \right] \psi,$$

like in Lorentz invariant theories, as their form comes from gauge invariance and therefore is insensitive to any Lorentz violation. Here we only consider chiral representations:

$$t_L^b \neq t_R^b, \quad \text{for some } b. \quad (2.3)$$

This is indeed the only case when anomalies can appear in Lorentz invariant theories.

All the ingredients introduced so far are also present in standard theories. We now want to write a (classical) action for ψ in the A_μ^b background, which involves Lorentz violating terms. By following the works of Coleman and Glashow [3] and Colladay and Kostelecky [4, 5], we assume the following properties:

- Locality,
- Translational invariance,
- Power-counting renormalizability (operators with dimension greater than four are not allowed).

These requirements tell us that the general form of the action is [5]

$$S = \int d^4x \left(i \bar{\psi} \Gamma^\mu D_\mu \psi - \bar{\psi} M \psi \right), \quad (2.4)$$

where we have adopted the signature $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$, Γ^μ and M are general constant 4×4 matrices:

$$\Gamma^\mu \equiv c^\mu_\nu \gamma^\nu + d^\mu_\nu \gamma_5 \gamma^\nu + e^\mu + i f^\mu \gamma_5 + \frac{1}{2} g^{\mu\nu\rho} \sigma_{\nu\rho}, \quad (2.5)$$

$$M \equiv m + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu} + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu. \quad (2.6)$$

Observe however that an additional term of the form $m' \gamma_5$ can be added to (2.6), but this may be removed from the action via a chiral transformation. Here γ^μ are the usual Dirac matrices, $\sigma_{\mu\nu} \equiv i[\gamma_\mu, \gamma_\nu]/4$ and we have introduced the Lorentz violating parameters

$$c^\mu_\nu - \delta^\mu_\nu, d^\mu_\nu, e^\mu, f^\mu, g^{\mu\nu\rho}, H^{\mu\nu}, a_\mu, b_\mu. \quad (2.7)$$

If the fermion representation is made by more than one irreps, the parameters in (2.7) are generically different for different irreps; we understand here an additional index labeling different irreps. The Lorentz violating perturbations that we have introduced can be divided into a *CPT-even* set

$$c^\mu_\nu - \delta^\mu_\nu, d^\mu_\nu, H^{\mu\nu}$$

and a *CPT-odd* one

$$e^\mu, f^\mu, g^{\mu\nu\rho}, a_\mu, b_\mu.$$

The experimental limits (for a recent summary of experimental constraints see [11]) require that, in a frame in which the earth is not relativistic, all the quantities in (2.7) are very small, in the sense that $c^\mu_\nu - \delta^\mu_\nu, d^\mu_\nu, e^\mu, f^\mu, g^{\mu\nu\rho} \ll 1$ and $H^{\mu\nu}, a_\mu, b_\mu \ll m$ [6]. In this paper we always work in such a frame. Also the parameters in (2.7) and m are real as a consequence of $S^\dagger = S$ (in our conventions $\bar{\psi} \equiv \psi^\dagger \gamma^0$ and $(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$).

Some consistency checks of this model (in the free field case, $A_\mu^b = 0$) have been performed in [6]. There it is shown that inconsistencies emerge at very high energies or equivalently in frames that move at very high speed with respect to earth-based laboratories. These energies (or equivalently boosts) are at a very high scale Λ where the spontaneous symmetry breaking of Lorentz invariance occurs. For example it is conceivable, but not obligatory, that Λ is the Planck scale. Therefore, the model at hand should be considered as a low energy effective description. From an effective field theory point of view we expect [6] $c^\mu_\nu - \delta^\mu_\nu, d^\mu_\nu, e^\mu, f^\mu, g^{\mu\nu\rho}$ to be at most of order m/Λ and $H^{\mu\nu}, a_\mu, b_\mu$ to be at most of order m^2/Λ and therefore these parameters are tiny if m is identified with the mass of the observed fermions.

In the following we will not assume $-\bar{\psi} M \psi$ to be invariant under (2.1); in this way our analysis will be applicable also to those theories, like the minimal SM, where the fermion masses emerge from the spontaneous symmetry breaking of a gauge symmetry. However, we do assume the first term in (2.4) to be invariant under (2.1) as, at least in the power-counting renormalizable case, the Higgs mechanism cannot modify that term. Since the generators satisfy (2.3), we have

$$\Gamma^\mu = c^\mu_\nu \gamma^\nu + d^\mu_\nu \gamma_5 \gamma^\nu, \quad (2.8)$$

which is also the most general form of Γ^μ compatible with the SM gauge group [5].

To study anomalies we introduce the functional $W[A]$ in the standard way, that is $\exp(i W[A]) \equiv \int \delta\psi \delta\bar{\psi} \exp(i S[A])$, with the normalization of the fermion measure chosen in

a way that $\exp(iW[0]) = 1$. As usual the absence of anomalies corresponds to the gauge invariance of $W[A]$ under (2.2):

$$\delta W[A] = 0 \quad + \quad M\text{-terms,} \quad (\text{in the absence of anomalies}), \quad (2.9)$$

where M -terms represent the non invariance of $W[A]$ due to non gauge invariant terms in $-\bar{\psi}M\psi$, if any. Condition (2.9) is equivalent to the Ward identities (WIs) for the n -point Green functions

$$\langle J_{b_1}^{\mu_1}(x_1) \dots J_{b_n}^{\mu_n}(x_n) \rangle = \int \delta\psi \delta\bar{\psi} \exp(iS[A=0]) J_{b_1}^{\mu_1}(x_1) \dots J_{b_n}^{\mu_n}(x_n), \quad (2.10)$$

like in the Lorentz invariant case. However, here we have to change the definition of the currents according to our classical action:

$$J_b^\mu \equiv \bar{\psi} \Gamma^\mu T_b \psi, \quad \text{with} \quad T_b \equiv t_L^b P_L + t_R^b P_R.$$

We now summarize the physical results of Ref. [10]. The WIs for (2.10) can be derived from the functional integral by assuming the invariance of the fermion measure. As usual the anomalies can be thought as a non-trivial Jacobian associated with a transformation of the form (2.1) and therefore in perturbation theory corresponds to a one-loop effect. For this reason we can restrict our attention to one-loop contributions. The presence of anomalies modifies the first term on the right-hand side of (2.9), which acquires a non vanishing value $\delta W[A]_{anom}$. Below we shall focus on the part $\delta W[A]_{anom}^{(2)}$ of this functional, which depends quadratically on A_μ^b . To understand the effect of Lorentz violations on the anomalies one should deal with the 3-point functions and compute the corresponding triangle graphs (that lead to $\delta W[A]_{anom}^{(2)}$). These diagrams involve the complete fermion propagator, which can be obtained by inverting the operator $i\Gamma^\mu D_\mu - M$ in (2.4), and the generalized Dirac matrices Γ^μ in the vertices. By performing this quantum computation in an explicit momentum cutoff regularization, one finds that the anomalous part of the WIs is independent of the Lorentz violating parameters in (2.8) and (2.6). Moreover, one can explicitly verify that the anomalous part of $\delta W[A]$ cannot be canceled by adding local counterterms (which corresponds to a change of the regularization) even if these counterterms violate Lorentz symmetry. Therefore, the anomaly cancellation conditions turn out to be remarkably stable under the Lorentz violating perturbations that we have considered.

3. A semiclassical argument

The results that we have just summarized come from a detailed quantum computation performed in Ref. [10]. Here we want to provide a simple argument which shortly illustrates the origin of the above-mentioned results. Such an argument uses some quantum results, like the symmetries of $\delta W[A]$, and some classical aspects, like the structure of the action in (2.4). We shall therefore refer to it as a semiclassical argument.

Also in the following we consider the case

$$m = 0 \quad \text{and} \quad H^{\mu\nu} = 0,$$

and so

$$M = a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu. \quad (3.11)$$

Indeed, any term in M , which involves an even number of Dirac matrices, does not contribute to the anomalies and to show this one can use an argument that leads to the m -independence of the anomalies in the Lorentz invariant case [10]. We therefore ignore m and $H^{\mu\nu}$ and refer to [10] for their explicit treatment.

Let us start by considering the anomalous part of $\delta W[A]$ in the Lorentz invariant case. This may be written [12] as follows:

$$\delta W[A]_{anom} = \frac{1}{48\pi^2} \text{Tr} \int d^4x \epsilon^{\mu\nu\lambda\rho} \Omega_L \partial_\mu \left(2A_\nu^L \partial_\lambda A_\rho^L - iA_\nu^L A_\lambda^L A_\rho^L \right) - (L \rightarrow R), \quad (3.12)$$

where we have defined

$$\Omega_{L(R)} \equiv \Omega^b t_{L(R)}^b, \quad A_\mu^{L(R)} \equiv A_\mu^b t_{L(R)}^b,$$

and $\epsilon^{\kappa\nu\lambda\rho}$ is the totally antisymmetric quantity with $\epsilon^{0123} = 1$. The part in (3.12) that is quadratic in $A_\mu^{L(R)}$,

$$\delta W[A]_{anom}^{(2)} = \frac{1}{24\pi^2} \text{Tr} \int d^4x \epsilon^{\mu\nu\lambda\rho} \Omega_L \partial_\mu A_\nu^L \partial_\lambda A_\rho^L - (L \rightarrow R), \quad (3.13)$$

can be computed by evaluating the 3-point functions [13] in a particular regularization, whereas the remaining terms can be obtained by using the Wess-Zumino consistency condition [14].

We would like to understand why the Lorentz violating deformations of the theory, corresponding to (2.8) and (3.11), cannot remove the anomalies altogether and to do so it is sufficient to focus on the quadratic functional in (3.13). Indeed, if it were possible to remove the anomalies, in particular there would be a way to cancel its bilinear part in $\delta W[A]$. Notice now that (3.13) is invariant under the following transformations:

1. Constant shifts of the gauge fields performed independently in the left-handed and right-handed parts,
2. General coordinate transformations performed independently in the left-handed and right-handed parts.

The first property is generically broken by the cubic terms in (3.12), whereas the second one is a well-known feature of the complete functional $\delta W[A]_{anom}$.

Meanwhile, the classical action in (2.4) can be expanded as follows:

$$\begin{aligned} S = & \int d^4x i \overline{\psi_L(x)} L_\nu^\mu \gamma^\nu \left[D_\mu + iL^{(-1)\rho}{}_\mu (a_\rho - b_\rho) \right] \psi_L(x) \\ & + \int d^4x i \overline{\psi_R(x)} R_\nu^\mu \gamma^\nu \left[D_\mu + iR^{(-1)\rho}{}_\mu (a_\rho + b_\rho) \right] \psi_R(x), \end{aligned} \quad (3.14)$$

where

$$L_\nu^\mu \equiv c_\nu^\mu - d_\nu^\mu, \quad R_\nu^\mu \equiv c_\nu^\mu + d_\nu^\mu$$

and $L^{(-1)\mu}{}_\nu$ and $R^{(-1)\mu}{}_\nu$ are the respective inverse matrices (which exist in our frame because the breaking of Lorentz invariance has to be small). Also, for later convenience, we have explicitly written the dependence of the fields on x . The only differences between (3.14) and its Lorentz invariant limit are therefore (i) two (generically) independent constant shifts of the gauge fields (due to CPT violating terms in the action) and (ii) two (generically) independent non singular linear and homogeneous transformations of γ^μ (which can be interpreted as coordinate transformations).

We can eliminate these differences in the final result for the anomaly by redefining the gauge fields and the space-time coordinates as follows. First introduce the new Lie-algebra valued vector fields

$$A_\mu'^L \equiv A_\mu^L - L^{(-1)\rho}{}_\mu (a_\rho - b_\rho), \quad A_\mu'^R \equiv A_\mu^R - R^{(-1)\rho}{}_\mu (a_\rho + b_\rho). \quad (3.15)$$

This redefinition serves to hide the CPT-odd parameters a_μ and b_μ : now the action can be written in the following way:

$$S = \int d^4x i \overline{\psi_L(x)} L^\mu_\nu \gamma^\nu D'_\mu \psi_L(x) + \int d^4x i \overline{\psi_R(x)} R^\mu_\nu \gamma^\nu D'_\mu \psi_R(x), \quad (3.16)$$

where

$$D'_\mu \psi_L(x) \equiv \left(\partial_\mu - i A'^L_\mu(x) \right) \psi_L(x) \quad \text{and} \quad D'_\mu \psi_R(x) \equiv \left(\partial_\mu - i A'^R_\mu(x) \right) \psi_R(x).$$

Then we consider the additional transformations of the coordinates and the gauge fields

$$\begin{aligned} L^\mu_\nu \frac{\partial}{\partial x^\mu} &\equiv \frac{\partial}{\partial x^\nu_L}, & L^\mu_\nu A'^L_\mu(x) &\equiv \tilde{A}^L_\nu(x_L), \\ R^\mu_\nu \frac{\partial}{\partial x^\mu} &\equiv \frac{\partial}{\partial x^\nu_R}, & R^\mu_\nu A'^R_\mu(x) &\equiv \tilde{A}^R_\nu(x_R), \end{aligned} \quad (3.17)$$

in a way that we can write

$$S = \int d^4x i \overline{\tilde{\psi}_L(x)} \gamma^\mu \left(\frac{\partial}{\partial x^\mu} - i \tilde{A}^L_\mu(x) \right) \tilde{\psi}_L(x) + (L \rightarrow R), \quad (3.18)$$

where

$$\tilde{\psi}_L(x) \equiv \sqrt{\det L} \psi_L(Lx) \quad \text{and} \quad \tilde{\psi}_R(x) \equiv \sqrt{\det R} \psi_R(Rx), \quad (3.19)$$

which is a sort of chiral transformation. We can see that the action assumes a Lorentz invariant form in terms of the new fields $\tilde{\psi}_L$, $\tilde{\psi}_R$, \tilde{A}^L_μ and \tilde{A}^R_μ . Therefore, we can derive the anomalies with a standard procedure and, in a certain regularization, obtain

$$\delta W[A]_{anom} = \frac{1}{48\pi^2} \text{Tr} \int d^4x \epsilon^{\mu\nu\lambda\rho} \Omega_L \partial_\mu \left(2 \tilde{A}^L_\nu \partial_\lambda \tilde{A}^L_\rho - i \tilde{A}^L_\nu \tilde{A}^L_\lambda \tilde{A}^L_\rho \right) - (L \rightarrow R) \quad (3.20)$$

(the detailed quantum computation of [10] shows that the possible non invariance of the fermion measure under (3.19) does not lead to corrections). This can be considered as a generalization of the discussion in [3], where the rotational invariant case has been studied. The fields $\{\tilde{A}^L_\mu, \tilde{A}^R_\mu\}$ and $\{A^L_\mu, A^R_\mu\}$ are related by transformations of the form **1** and **2**, so

$$\delta W[A]^{(2)}_{anom} = \frac{1}{24\pi^2} \text{Tr} \int d^4x \epsilon^{\mu\nu\lambda\rho} \Omega_L \partial_\mu A^L_\nu \partial_\lambda A^L_\rho - (L \rightarrow R) + \dots, \quad (3.21)$$

where the dots represent additional bilinear terms coming from the cubic terms in (3.20), which are not invariant under constant shifts of the gauge fields. These additional bilinear terms contain only one derivative and therefore cannot help to cancel (3.13), which instead has two derivatives. Not even a general change of regularization, which does not necessarily assume Lorentz invariance, can change the fact that $\delta W[A]^{(2)}_{anom}$ is non-vanishing if it is so in the Lorentz invariant limit [10].

4. Conclusions

In this article we have summarized the discussion about the effect of Lorentz violation on the perturbative non-gravitational anomalies, which has been performed in Ref. [10]. Following the SME, we have assumed locality, translational invariance and power counting renormalizability and focused on a single family of fermions. Although the *anomaly functional* $\delta W[A]_{anom}$ can assume a more general form (Lorentz violating counterterms are allowed), the standard anomaly

cancellation conditions turn out to be necessary also in the presence of Lorentz violation. Moreover, here we have provided a simple and hopefully illuminating semiclassical argument, which explains the origin of this fact. This relies on two main points. The first one is the analysis of the differences between the classical structure of the action in the Lorentz invariant and Lorentz violating setups (in the latter case the parameters $L^\mu_\nu, R^\mu_\nu, a_\mu$ and b_μ are turned on). The second one is the observation of the symmetries of $\delta W[A]^{(2)}_{anom}$, the quadratic part of the anomaly functional.

An interesting development of this work may be the extension to anomalies that do not correspond to purely internal symmetries, like the gravitational anomalies. Since Lorentz transformations are particular general coordinate transformations, the breaking of Lorentz symmetry should occur spontaneously in the context of gravitational theories, triggered by the vacuum expectation value of tensors [15]. We therefore expect the generalization to gravitational anomalies to be non-trivial.

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References

- [1] *A very partial list is* R. M. Wald, “Quantum Gravity And Time Reversibility,” Phys. Rev. D **21** (1980) 2742. V. A. Kostelecky and S. Samuel, “Spontaneous Breaking of Lorentz Symmetry in String Theory,” Phys. Rev. D **39** (1989) 683. V. A. Kostelecky and S. Samuel, “Phenomenological Gravitational Constraints on Strings and Higher Dimensional Theories,” Phys. Rev. Lett. **63** (1989) 224. V. A. Kostelecky and S. Samuel, “Gravitational Phenomenology In Higher Dimensional Theories And Strings,” Phys. Rev. D **40** (1989) 1886. V. A. Kostelecky and S. Samuel, “Photon and Graviton Masses in String Theories,” Phys. Rev. Lett. **66** (1991) 1811. V. A. Kostelecky and R. Potting, “CPT and strings,” Nucl. Phys. B **359** (1991) 545. T. Jacobson, S. Liberati and D. Mattingly, “Quantum gravity phenomenology and Lorentz violation,” Springer Proc. Phys. **98** (2005) 83 [arXiv:gr-qc/0404067].
- [2] O. Bertolami, D. Colladay, V. A. Kostelecky and R. Potting, “CPT violation and baryogenesis,” Phys. Lett. B **395** (1997) 178 [arXiv:hep-ph/9612437]. V. A. Kostelecky and M. Mewes, “Lorentz and CPT violation in neutrinos,” Phys. Rev. D **69** (2004) 016005 [arXiv:hep-ph/0309025]. M. Libanov, V. Rubakov, E. Papantonopoulos, M. Sami and S. Tsujikawa, “UV stable, Lorentz-violating dark energy with transient phantom era,” JCAP **0708** (2007) 010 [arXiv:0704.1848 [hep-th]].
- [3] S. R. Coleman and S. L. Glashow, “High-energy tests of Lorentz invariance,” Phys. Rev. D **59** (1999) 116008 [arXiv:hep-ph/9812418].
- [4] D. Colladay and V. A. Kostelecky, “CPT violation and the standard model,” Phys. Rev. D **55** (1997) 6760 [arXiv:hep-ph/9703464].
- [5] D. Colladay and V. A. Kostelecky, “Lorentz-violating extension of the standard model,” Phys. Rev. D **58** (1998) 116002 [arXiv:hep-ph/9809521].
- [6] V. A. Kostelecky and R. Lehnert, “Stability, causality, and Lorentz and CPT violation,” Phys. Rev. D **63** (2001) 065008 [arXiv:hep-th/0012060].
- [7] V. A. Kostelecky, C. D. Lane and A. G. M. Pickering, “One-loop renormalization of Lorentz-violating electrodynamics,” Phys. Rev. D **65** (2002) 056006 [arXiv:hep-th/0111123].
- [8] G. de Berredo-Peixoto and I. L. Shapiro, “On the renormalization of CPT/Lorentz violating QED in curved space,” Phys. Lett. B **642** (2006) 153 [arXiv:hep-th/0607109]. D. Colladay and P. McDonald, “One-loop renormalization of pure Yang-Mills with Lorentz violation,” Phys. Rev. D **75** (2007) 105002 [arXiv:hep-ph/0609084]. D. Anselmi and M. Halat, “Renormalization of Lorentz violating theories,” Phys. Rev. D **76** (2007) 125011 [arXiv:0707.2480 [hep-th]]. D. Colladay and P. McDonald, “One-Loop Renormalization of QCD with Lorentz Violation,” Phys. Rev. D **77** (2008) 085006 [arXiv:0712.2055 [hep-ph]].
- [9] *Some examples are* R. Jackiw and V. A. Kostelecky, “Radiatively induced Lorentz and CPT violation in electrodynamics,” Phys. Rev. Lett. **82** (1999) 3572 [arXiv:hep-ph/9901358]. M. Perez-Victoria, “Physical (ir)relevance of ambiguities to Lorentz and CPT violation in QED,” JHEP **0104** (2001) 032 [arXiv:hep-th/0102021]. B. Altschul, “Failure of gauge invariance in the nonperturbative formulation of massless Lorentz-violating QED,” Phys. Rev. D **69** (2004) 125009 [arXiv:hep-th/0311200].

- [10] A. Salvio, “Relaxing Lorentz invariance in general perturbative anomalies,” *Phys. Rev. D* **78** (2008) 085023 [arXiv:0809.0184 [hep-th]].
- [11] V. A. Kostelecky and N. Russell, “Data Tables for Lorentz and CPT Violation,” arXiv:0801.0287 [hep-ph].
- [12] B. Zumino, Y. S. Wu and A. Zee, “Chiral Anomalies, Higher Dimensions, And Differential Geometry,” *Nucl. Phys. B* **239** (1984) 477.
- [13] J. S. Bell and R. Jackiw, “A PCAC puzzle: $\pi^0 \rightarrow \gamma \gamma$ in the sigma model,” *Nuovo Cim. A* **60** (1969) 47. S. L. Adler, “Axial vector vertex in spinor electrodynamics,” *Phys. Rev.* **177** (1969) 2426.
- [14] J. Wess and B. Zumino, “Consequences of anomalous Ward identities,” *Phys. Lett. B* **37** (1971) 95.
- [15] V. A. Kostelecky, “Gravity, Lorentz violation, and the standard model,” *Phys. Rev. D* **69** (2004) 105009 [arXiv:hep-th/0312310].